

## Realistic Mathematics Education

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### Introduction

Mathematics serves as a significant tool enabling us to delve into the depths of knowledge and understanding as a universal language. Mathematics is a discipline that is not exclusive only to scientists; it is a field that individuals consistently require in their daily lives. However, for many individuals, mathematics can be perceived as a world filled with abstract concepts and complex formulas. Realistic Mathematics Education aims to connect mathematical concepts to real-life contexts for students and to make sense of this discipline through concrete experiences. This book chapter aims to provide a detailed overview of the Realistic Mathematics Education (RME) teaching theory.

In the process of developing mathematical thinking, the connections students establish with the world they live in can transform knowledge into a tool that is usable throughout their lives rather than just a learned concept. While the ability to perform rapid calculations and apply mathematical formulas and rules quickly may not be as crucial as before, individuals are now expected to effectively solve problems, engage in logical reasoning, and adapt to new situations (Birgin and Tutak, 2006). Furthermore, mathematics education has various benefits such as enhancing creative thinking, reasoning skills, and the ability to appreciate aesthetics in an individual (Gür, 2006). Freudenthal (2012) expressed the connection between mathematics and real life as "mathematics is life itself, arising from real-life problems".

From the past to the present, many theories have been developed in the field of mathematics education. Realistic Mathematics Education (RME) is a relatively recent teaching theory that began to take root in the 20th century. Various definitions exist regarding this approach.

According to Van den Heuvel-Panhuizen (2000), within the framework of the Realistic Mathematics Education (RME) theory, students generate meaningful learning in their minds by establishing a bridge between informal and formal knowledge through addressing real-life problems. The informal knowledge is gradually transformed into formal knowledge in the student's cognitive processes.

Mathematics should include real-life experiences and all imaginative constructs that an individual can visualize in their mind. The word "realistic", which forms the name of the RME approach, covers not only real life but also everything real and imaginary. Freudenthal (1968), on the other hand, defined this approach as an in-depth exploratory process involving recognizing a problem situation, solving the problem, generating a subject, reshaping that subject, and making it meaningful by concretizing it. Freudenthal argued against the traditional method in mathematics education, considering it anti-didactic to present formal knowledge created by rules first and then engage in application. Instead, he asserted that real life is no different from mathematics and that formal knowledge can be reached by starting from real-life problems.

### **History of Realistic Mathematics Education**

After the establishment of the foundations of Realistic Mathematics Education (RME), this innovative movement triggered a significant transformation in mathematics education in the Netherlands. Developed in the mid-20th century under the leadership of Hans Freudenthal, this teaching theory is based on the intuitionist philosophy of mathematics and was shaped under the influence of Jan Brouwer, who was Freudenthal's student (Iemhoff, 2014). The theoretical foundations of RME gained a formal structure with the "Wiskobas Project" initiated by Wijdeveld and Goffree in 1968, and was supported by the institute called "The Instituut Ontwikkeling Wiskundeonderwijs" (IOWO) in 1971 under the leadership of Hans Freudenthal. The initiation of the Wiskobas Project and the establishment of IOWO aimed to launch a national mathematics education reform with innovations in mathematics teacher training (Van den Heuvel-Panhuizen and Drijvers, 2020). The Wiskobas Project aimed not only to reform mathematics education in the Netherlands but also to bring about innovations in international mathematics education (Cansız, 2016).

The RME approach, proposed by Hans Freudenthal in the 1970s as a reaction to the traditional approach, has been shaped around Freudenthal's views on mathematics education. For Freudenthal, mathematics is a human activity and cannot be abstracted from real life (Zulkardi, 2000). One of Freudenthal's approaches to creating the RME theory is the idea that people will use mathematics to solve everyday problems in the future (Gravemeijer and Terwel, 2000). This educational approach, which emerged in response to the existing understanding of mathematics education, has been accepted in various countries such as the United States, Japan, the United Kingdom, Germany, Malaysia, Denmark and incorporated into the process (Çetin, 2018). As an alternative to this, 'New Math', that is, the structuralist approach, was being discussed instead of the traditional understanding. Dutch educator and mathematician Hans Freudenthal advocated for the modernization of education and moved beyond the structuralist approach, establishing the unique principles for realistic mathematics education. The foundations of realistic mathematics education were laid by the Wiskobas Project, with the involvement of Adri Treffers (Van den Heuvel-Panhuizen, 2014).

RME is a teaching method specific to the field (mathematics education) (Treffers, 1987; Gravemeijer, 1994; Van den Heuvel-Panhuizen, 1998). The significance of RME is the considerable inclusion of a variety of "realistic" situations in the learning process. The mentioned realistic situations serve as a source to initiate the

development of mathematical concepts and tools, providing a structure for students to apply their mathematical knowledge in later stages. This structure gradually becomes more formal and general over time. According to Gravemeijer (1999), in RME, the term "realistic" is crucial when used in the sense of "real-world" situations, yet in this context, the concept of "realistic" has a broader connotation. In RME, the concept of "realistic" means presenting students with problem situations they can imagine. The Dutch equivalent of the English word "realistic" in RME is "zich realiseren", that is, to imagine, to comprehend (Alacacı, 2016; Topbaş Tat, 2020). Therefore, the problems presented in RME can come from the real world, but they can also originate from the fantasy world of fairy tales or the formal world of mathematics as long as they are experientially real in the student's mind.

### **Mathematization**

One of the fundamental concepts of RME is Freudenthal's (2012) idea of mathematics as a human activity. For Freudenthal, mathematics was not the body of mathematical knowledge, but the activity of solving problems and looking for problems, and, more generally, the activity of organizing a subject from reality or mathematical subject which he called 'mathematization' (Heuvel-Panhuizen, 2003). Freudenthal (2012) explained very clearly what mathematics is about with the following statements: "There is no mathematics without mathematizing". This activity-based interpretation of mathematics has had significant implications for how mathematics education is conceptualized.

Here, it is to reveal and enrich mathematical knowledge through a gradual process of mathematization (Alacacı, 2016; Topbaş Tat, 2020). Treffers (1987) expressed the way of mathematization in two ways: these are horizontal and vertical mathematization. This statement brought about a change in what is known about mathematization. Freudenthal (1991) examined horizontal mathematization as transitioning from real life to the world of symbols, while he expressed vertical mathematization as movements in the world of symbols. During this process, students became aware of solution methods, while simultaneously exploring relationships between concepts, and could make practical applications (Van den Heuvel-Panhuizen, 2003). However, in RME, horizontal and vertical mathematization should complement each other. Because, for mathematization to occur, first horizontal and then vertical mathematization are targeted.

### **Horizontal and Vertical Mathematization**

Horizontal mathematization is defined as the process of mentally visualizing a given real-life problem, formalizing it, and transferring real-life problems to mathematical problems (Van den Heuvel-Panhuizen, 2000). Horizontal mathematization is not limited to a specific process or stage in teaching; what matters here is that the selected real-life problems are applicable (Atasoy, 2017).

During the process of horizontal mathematization, students are expected to:

- Use mathematical tools in solving or organizing problems related to real-life situations
  - Express and define mathematics within a context
  - Visualize and formalize the problem in different ways
  - Explore and express relationships within real-life problems and transform the problem into a mathematical one (Üzel, 2007).

Vertical mathematization, on the other hand, is the process of relating and organizing within the mathematical system using the findings or models that students have discovered at the end of the horizontal mathematization process (Van den Heuvel-Panhuizen, 2003). Being able to prove, making adjustments to models to make them suitable for the situation, and generalizing them are the most evident characteristics of vertical mathematization (Zulkardi, 1999). Vertical mathematization leads to a structuring within the mathematical system, and students review the situation with their own activities and interpretations (İnce, 2019). Here, while the individual activities of students are important, individuals achieve a higher level of mathematical understanding at the end of the vertical mathematization process.

During the vertical mathematization process, mathematical systems are reevaluated within their own framework. In this process, students are expected to:

- Express a relationship in a formula,
- Internalize and adapt example situations and provide different examples,
- Combine examples and generalize them by formalizing (Üzel, 2007).

Horizontal and vertical mathematization is given in Figure 1.

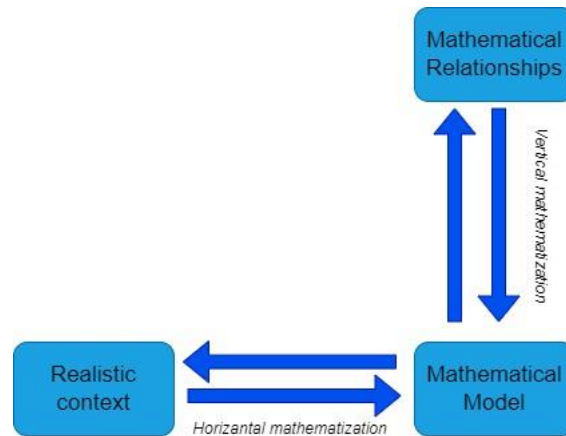


**Figure 1.** Horizontal and Vertical Mathematization (Özdemir and Üzel, 2013)

When Figure 1 is examined, the process of mathematization begins with the creation of a real context experimentally, and it continues with the creation of a mathematical model, which is termed as horizontal mathematization. A framework for mathematical relationships is established through mathematical models, which is termed as vertical mathematization. Similarly, starting from the framework of mathematical relationships, the same process can be experienced. The process of mathematization progresses gradually, and the first stage is completed with horizontal mathematization. The second stage continues with vertical mathematization and concludes. In other words, the necessary and sufficient condition for being able to perform vertical mathematization is the completion of horizontal mathematization (Freudenthal, 1983). According to De Lange (1987), on the other hand, performing horizontal mathematization is not a prerequisite for vertical mathematization; the mathematization process can take different paths.

### Basic Principles for the Design Process of Realistic Mathematics Education

Gravemeijer (1994, 1999) defines three fundamental intuitive principles for the RME teaching design process as follows:

- Guided Reinvention
- Didactical Phenomenology
- Self Developed or Emergent Models (Barnes, 2004).

The detailed headings related to these principles are presented below.

### **Guided Reinvention**

The basis of this principle is the process of reaching formal knowledge with the student's own mental power, based on real-life experiences. According to Freudenthal (2012), guided reinvention is the process in which individuals create unique solution paths and discover formal mathematical knowledge for themselves when confronted with real-life situations. It is not possible for the student to reach all formal knowledge through their own mental power alone (Freudenthal, 1991). Here, the main purpose of the guided reinvention principle is not just to make the concept discovered but to use the discovery as a tool to bridge informal knowledge with formal knowledge (Gravemeijer and Doorman, 1999).

Students should be guided in the process of reaching formal knowledge, and realistic problem situations involving activities and materials should be provided to help them bridge informal knowledge with formal knowledge (Kwon, 2002). In the process of reaching formal knowledge, the guide should be instructive in the use of logical solutions and steering away from incorrect methods (Drijvers, 2003). Thus, the reinvention process that occurs with a guide is defined as guided reinvention (Freudenthal, 1991).

Gravemeijer (1994) has modeled the stages of guided reinvention, and this model is presented in Figure 2.

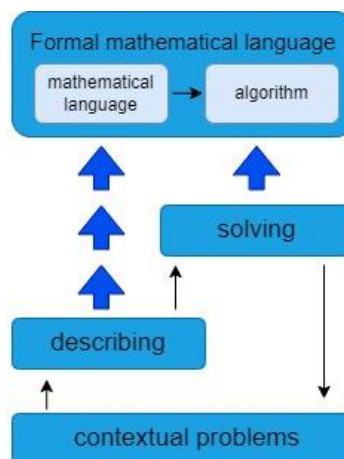


**Figure 2.** Guided Reinvention Model (Gravemeijer, 1994)

It is to search for or provide contexts that are experimentally real for students and can be used as starting points for gradual mathematization. In the early stages of RME, the principle of reinvention inspired from references to the history of mathematics. Later, this has evolved into accepting students' informal solution methods as a source of inspiration. Streefland (1991) explained this situation as sometimes the informal ways students follow while solving applied problems can anticipate the more formal mathematics targeted by the teacher. For example, while solving problems like dividing three pizzas among four children, students might be

observed to reach more mathematically grounded solution methods, such as adding and subtracting with fractions with unequal denominators (Gravemeijer, 1994).

### *Didactical Phenomenology*

Didactic phenomenology aims to explain the process of how concepts form by analyzing it (Kwon, 2002). Stimulating sources that enable the rediscovery of concepts are environmental problems. In other words, the primary source that causes the formation of mathematical concepts is real-life situations, as it was in history. For the beginning of the rediscovery of concepts, it is important to present individuals with problem situations that they can eventually generalize (Gravemeijer, Heuvel, and Streefland, 1990). According to Treffers and Goffree (1985), this principle should fulfil four functions:

- **Concept Formation:** It should enable students to access mathematics in a natural and motivating way.
- **Model Formation:** It should provide a solid foundation for thinking and learning formal operations, procedures, and rules alongside other models.
- **Applicability:** It should use reality as a source and field of application.
- **Application:** It should make use of students' specific skills in applied areas (Barnes, 2004).

Didactic phenomenology describes how mathematical knowledge should be formed in the student. Gravemeijer (1994) states that the aim of didactic phenomenology principle is to find real-life situations that can serve as a source for the generalization of concepts, contributing to the vertical mathematization process. The term "real" here may not necessarily refer to commonly encountered situations but rather to events that individuals can construct in their own minds (Nelissen, 1999). This process is expressed in the student through the term "gradual mathematization," consisting of horizontal and then vertical components (Alacacı, 2016).

For instance, to construct length as a mathematical object, students should be exposed to situations that require organizing facts based on length (Gravemeijer, 1994). In the framework of didactic phenomenology, situations applying a specific mathematical topic should be investigated to assess their suitability as impact points for the advanced process of mathematization. Considering the historical evolution of mathematics from practical problem-solving, it would make sense to seek problems in contemporary applications that lead to this process. Later, it can be said that formal mathematics emerged through the generalization and transformation of problem-solving methods and concepts developed specifically for these identified situations into formalized mathematics (Gravemeijer and Tervel, 2000). According to the didactic phenomenology principle, there are two different phenomenological situations for a mathematical concept. These are mathematical phenomenology and real-life phenomenology. In mathematical phenomenology, the goal is to explain the mathematical structure of a concept to the individual and to explain the problems that may arise in the process of concept formation. In the real-life phenomenology, the aim is to explain the mathematical structures within a real-life situation and investigate whether it enhances or hinders an individual's conceptual understanding (Oldham et al., 1999).

### ***Self Developed or Emergent Models***

This principle targets models that students develop on their own. In the rediscovery of concepts, individuals reach a formal understanding through modeling real-life situations and using these models (Gravemeijer, 1999). In other words, the models developed by the individual serve as a bridge between informal knowledge and formal knowledge.

During the mathematization process, students are presented with real-life situations, and the student, without being exposed to external influences, comprehends and models this situation. Later, this model is generalized to extend beyond just that particular problem and becomes applicable to similar situations (Treffers, 1991).

When planning teaching, educators should choose problem situations that allow students to comprehend and model effectively. Thus, the student, by starting from situations they can model, completes their own process of learning mathematics. For example, in elementary school mathematics textbooks, the teaching of the number line is directly introduced by drawing the shape, sometimes referencing models such as a garden fence. According to Freudenthal, the founder of RME, all mathematical concepts emerge through individuals mathematizing their real-life experiences (Gravemeijer, 1990). Therefore, the teaching of the number line can also occur through mathematizing a real situation. Without doing so, teaching should not be approached with an attitude like "this is called a number line" (Altun, 2006).

### **Key Principles of Realistic Mathematics Education**

Treffers (1991) summarized the fundamental principles of the RME approach under five headings. These principles are listed below.

- Creation and concretization
- Levels and models
- In-depth thinking and special assignments
- Social context and interaction
- Structuring and collaborative processing

These principles are explained in detail in this section.

***Creation and Concretization:*** According to this principle, instruction should begin by addressing a concrete concept. The knowledge should be taught through a constructivist approach rather than being directly transmitted in the learning process. Exactly, the individual should undergo the process of learning mathematics by starting from concrete concepts instead of receiving ready-made knowledge. The fundamental element necessary for initiating the student's learning process is real problems encountered, and through these problems, the student is motivated to find themselves within the process of learning mathematics (Gravemeijer, 1994; Barnes, 2004).

According to Freudenthal (1999), real-life situations are not just tools that conclude the learning process but rather sources that start and maintain the process. These real-life situations should be selected logically and in accordance with the mathematical structure for students to be able to visualize them in their minds and reveal their prior knowledge (Van den Heuvel-Panhuizen and Wijers, 2005). As a concrete tool, real-life situations hold a significant place in the RME approach.

Because the student observes aspects of their own life in solving real-life problems, they use it as a motivational tool (Gravemeijer and Doorman, 1999).

**Levels and Models:** Modeling informal knowledge in the mathematization process makes it easier to access formal knowledge. Continuing through the process by creating a model allows the individual to make a comprehensible formalization (Treffers, 1991).

Gravemeijer (1994) has illustrated the process of creating a model in four levels: situational level, reference level, general level, and formal level.

1. Situational Level: At this level, individuals use their informal knowledge and strategies that are relevant to the given problem situation.
2. Reference Level: Individuals at this level create a model by referencing the presented problem situation.
3. General Level: At this stage, individuals generalize the created model. They develop the model to use it in different situations.
4. Formal Level: At this level, individuals engage with traditional rules. It represents the formal level of mathematics.

Swetz and Hartler (1991) have articulated the modeling process in four stages. These stages are outlined below and schematized in Figure 3.

1. Observation and perception of the problem situation.
2. Identifying factors affecting the problem and determining the relationship between factors.
3. Transferring the conducted analyses into a model.
4. Testing the suitability of the obtained result for the problem and reinterpreting it.

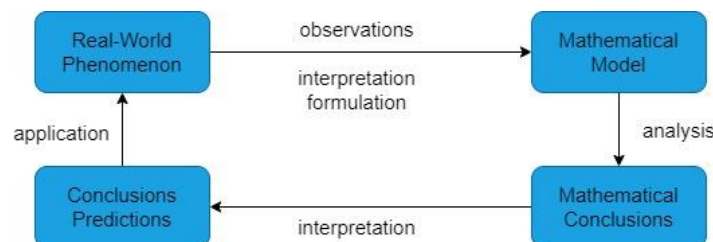


**Figure 3.** Modeling Process (Swetz and Hartler, 1991)

**In-Depth Thinking and Special Assignments:** In-depth thinking can be defined as an idea put forward by reflecting on one's own thoughts or the thoughts of others (Memnun, 2011). This principle emphasizes the importance of the production carried out by the individual. Therefore, De Lange (1995) argued that individuals should be given assignments such as conducting experiments, writing articles, and making inferences from experiments in order to enhance their productivity.

According to the RME approach, students should be motivated to develop strategies on their own and produce a product, and opportunities should be provided for this (Widjaja and Heck, 2003). With these opportunities provided, it becomes easier for students to construct knowledge.

**Social Context and Interaction:** This principle constitutes a significant portion of the RME approach in terms of student-teacher and student-student interaction.

Because the RME approach describes learning mathematics as a social activity. Just like in the traditional approach, not only student-teacher interaction but also student-student interaction is crucial for this approach (Treffers, 1991).

The RME approach recognizes that all students can't learn at the same pace simultaneously. Therefore, it organizes learning by grouping individuals who are learning simultaneously into interactive groups. Thus, during the learning process, they build a sense of self-confidence as they realize their own learning through interaction with each other (Van den Heuvel-Panhuizen and Wijers, 2005; Nelissen and Tomic, 1993).

**Structuring and Collaborative Processing:** According to the RME approach, providing information ready-made to students is an unacceptable concept. According to RME, the student structures knowledge in their mind based on real situations (Treffers, 1991).

Mathematics is a whole, and treating topics separately means ignoring the relationship between them. Therefore, the integrity of the topics is used in solving real-life problems. The topics should be taught in an integrated manner, emphasizing their interconnectedness, rather than addressing them separately. In this way, students structure the connection between mathematical concepts by processing topics together in their minds (Zulkardi, 1999).

### Teaching and Learning Principles of Realistic Mathematics Education

As mentioned in the previous section, Treffers considered the RME approach as five principles, and Van den Heuvel-Panhuizen (2000), who has contributed to numerous studies in mathematics education, has developed these principles. Van den Heuvel-Panhuizen and Wijers (2005) explained these principles under six headings in their study.

#### Principle of Activity

Based on Freudenthal's statement "*Mathematization is a learning activity.*", the student is at the very center of the learning process and realizes their own learning. Students are confronted with real-life problems, they develop strategies to solve the problem and thereby access the knowledge.

In this principle, the individual does not take what is ready; instead, they actively engage in the process to produce an outcome. Taking this principle into account, during the teaching process, the student should be active within the lesson and not left in a passive state. Otherwise, the ready-made information that the student passively receives loses its instructive quality (Freudenthal, 1968; 2012).

#### Principle of Reality

The aim of mathematics education is to comprehend mathematical concepts and the systematics of mathematics, to reflect them in real life, and to acquire fundamental mathematical skills for education and advancement in a specific field (MONE, 2018). The RME approach also serves this purpose by using reality as a source. The

important thing here is to present real situations that the student can make sense of and to enable him to access the information himself.

The RME approach suggests that solving a problem situation does not begin by using ready-made and abstract mathematical knowledge (Wubbels et al., 1997). In RME, there is a process of starting from the real and reaching the abstract. In this context, real can refer to either a real-life situation or a fiction.

### **Principle of Level**

The process of mathematization involves progressing through specific levels, from understanding and modeling problem situations to interpreting models and reaching formal mathematical knowledge. Without completing the first level, it is not possible to transfer what is done at the first level to the second level.

Models have an important place in this process. After transforming the problem situation into informal knowledge, models serve as a bridge until formal knowledge is reached. A specific model created for a situation should be generalized to be applicable to all similar situations (Van den Panhuizen-Heuvel and Wijer, 2005).

This principle is important for establishing a mathematical perspective and ensuring consistency in the development of the mathematics program (Memnun, 2011).

### **Principle of Interrelationship**

Since mathematics has an integrated structure, each mathematical topic cannot create a meaningful community on its own. The RME approach also advocates teaching mathematics as a whole without breaking it down into parts. It is important for the student to have knowledge about multiple subjects while solving a problem for developing a better strategy for the problem. In other words, in the process of mathematization, the student needs to have knowledge about interconnected subjects to comprehend the information.

Freudenthal (2012) expressed that presenting mathematical topics independently would result in the student's inability to establish connections between the pieces of information and a faster tendency to forget. When preparing the mathematics curriculum, this principle of the RME should be considered, and mathematical topics should be organized in a spiral structure.

### **Principle of Interaction**

Van den Heuvel-Panhuizen and Drijvers (2014) defined the process of learning mathematics as a social activity. According to this definition, in the process of learning mathematics, students should be in a social environment, where they can share their thoughts, strategies, and solutions with each other. This way, students, by seeing different thoughts and strategies from their own, realize that there is not only one way to solve a problem situation. They can develop their own strategies by making comments on different strategies.

In an environment where the whole class shares with each other without any level distinction, everyone feels productive, and all individuals become motivated in the

learning process. Different understanding levels should be addressed in the same environment without internal grouping, and problem situations that appeal to different levels of understanding should be presented for the realization of the teaching process as a whole (Van den Heuvel-Panhuizen, 2000).

Classroom environments should be created in the learning process where all individuals can interact with each other, based on this principle of the RME.

### **Principle of Guidance**

Freudenthal (1991) said that the individual should be guided in the process of rediscovering mathematics. In the process of guiding the individual, situations that will leave them passive should be avoided. Because Freudenthal described the mathematization process as a situation in which the individual is active. Rather than telling the individual what information to acquire, the teacher should guide them on how to access the information.

The teacher should create a suitable environment for the student in the process of learning mathematics, and carefully identify the source that initiates the mathematization process. An inadequately identified source may cease to be guiding in initiating the process.

### **Lesson Design Suitable for Realistic Mathematics Education**

Streefland (1991) stated that the design of lessons appropriate for RME should take place in three stages. These three stages are the classroom level, the lesson level, and the theoretical level. These stages are examined under the headings below.

#### **Classroom Level**

This level aims to carry out the horizontal mathematization process, which is one of the mathematization processes. Therefore, in the first stage of this level, a realistic life situation is presented to the student, where they can comprehend and reach a product. Then, this realistic situation is related. In this way, students are provided with the opportunity to create symbols, graphs, or models for the problem situation. In this process, an interactive classroom environment is provided where students can actively participate, express their thoughts, and be open to different perspectives. As a result of their interactions, tasks are assigned to students to ensure the continuity of the process and to enable them to present their own products (Zulkardi, 1999). Below, the classroom level modeling process is provided in Figure 5.

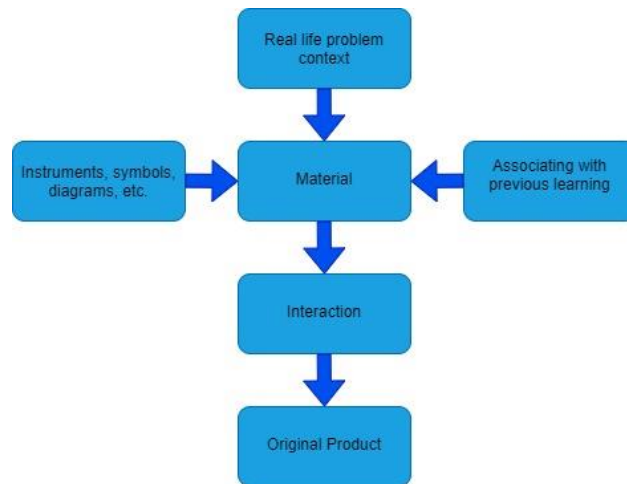


**Figure 5.** Classroom Level Lesson Modeling Process (Özdemir, 2020)

### Lesson Level

This level is a preparatory stage for transitioning to the theoretical level. The materials presented at the classroom level are used in the teaching process according to the instructional purpose of the lesson. After these materials are tested at the classroom level, it is expected that they will be further developed and used at the general level (Zulkardi, 1999).

### Theoretical Level

The theoretical level, unlike the classroom level, aims for vertical mathematization. The materials presented in the previous two levels and instructional products serve as resources for this level. By using the materials and products generated in the previous two levels, a general theory is developed. This theory is then tested and developed on different concrete models.

In the Realistic Mathematics Education approach, the following four components are considered to create a lesson plan.

#### 1. Goals

In mathematics education, according to the three target levels identified by De Lange (1995), the goals of traditional mathematics teaching are considered as lower-level, while the goals set by the RME approach are described as middle and upper-level goals. The reason traditional mathematics teaching goals are considered lower-level is that they consist of theoretical knowledge and definitions. In the RME approach, skills such as establishing relationships between concepts are considered middle-level goals, while goals like reasoning, strategy development, model creation, and producing a product are classified as upper-level (Zainurie, 2007). Realistic mathematics teaching suggests that lesson design should consist of middle and upper-level goals.

#### 2. Materials

The mathematics teaching program should be designed in a way that allows students to discover for themselves, represent information with concrete models, and learn by doing and experiencing (Bulut, 2004). Materials have an important place in this process. Concrete materials enable students to think and provide them with the

confidence to solve problems (Brecht, 2000). De Lange (1996) proposed the idea that materials should be related to real life for students to comprehend. These materials should be associated with real-life problems that serve as a source for the mathematization process.

### 3. Activities

In the RME approach, the teacher has a major role in the classroom. This role is shaped not as a transmitter of information, but as a deliverer of information. In all stages of the mathematization process, the teacher guides the student through the following steps.

- It provides students with real-life situations to serve as the source for the mathematization process,
- Enables students to establish relationships between concepts by providing hints for making sense of the situation,
- Creates an environment for students to interact with each other, share their thoughts,
- Provides opportunities for students to apply their solutions,
- Offers different contextual problems to students to generalize their problem-solving approaches.

In this stage, guidance actively plays a role in the process as a motivational tool for the student.

### 4. Evaluation

According to the Realistic Mathematics Education approach, evaluation should be made throughout the process, not just at the end. The teacher may ask students to write compositions, practice, and conduct experiments for observation throughout the process to assess their progress. The tasks performed by the students assist the teacher in planning for the next lesson and designing the lesson. At the same time, the assessment conducted during the teaching process encourages idea and strategy sharing among students, allowing them to develop their own strategies. In other words, evaluation during the teaching process is also instructive.

De Lange (1995) mentioned five basic features for evaluation. These features are given below.

- Evaluation aims to improve learning and teaching. In other words, evaluation should be actively conducted throughout the learning process to measure students' progress.
- Evaluation stages should allow for understanding what the student knows or doesn't know about a concept.
- Evaluation should actively engage all levels of mathematical goals (lower, middle, upper).
- Evaluation should include objective tests and problems that can measure whether the student has truly learned.
- Evaluation tools should be easily accessible and compatible with the structure and culture of the teaching environment.

In this case, the development of evaluation tools must be important. The stages of developing evaluation tools are given below.

- Contexts appropriate to the real situation are selected, starting from a source that students can make sense of.
- Relationships are established between previously learned concepts and those currently being learned.
- Following the activity principle, students interact to create thoughts based on each other's ideas and develop new models and materials.
- Opportunities are provided for students to mathematize based on the models they develop through interaction with each other.
- Evaluation materials should consist of open-ended questions that can lead students to original products (Zulkardi, 1999).

## **The Relation of Realistic Mathematics Teaching to Other Learning Approaches**

### **Realistic Mathematics Teaching and Constructivism**

Constructivist learning, which focuses on how knowledge is acquired, is distinct from teaching theories. In contrast, the RME is a teaching theory. Realistic Mathematics Education is compatible with constructivism, but it differs in the way knowledge is structured. While Realistic Mathematics Education rejects the separation of theoretical knowledge from application, constructivism does not reject this (Panhuizen 2001). In constructivism, the acquisition of knowledge is based on concrete materials and informal knowledge in accordance with the constructivist theory. The main distinction between Realistic Mathematics Teaching and constructivism lies in how the teaching begins. Realistic Mathematics Education always begins with contextual problems, while in the constructivist approach, the situation is not always the same. For teachers and experts to effectively implement Realistic Mathematics Education, they need to have a good understanding of contextual problems and be able to distinguish them from other types of problems.

### **Realistic Mathematics Education and Learning Through Discovery**

Demirel (2005) defined learning through discovery as the process of exploring the learning material by using existing knowledge without presenting the learning material in its final form. Jacobsen lists the steps of learning through discovery as follows:

- 1) Teacher presenting examples,
- 2) Students explaining the examples,
- 3) Teacher providing additional examples, ... etc. (Senemoğlu, 1997).

As can be seen, in discovery teaching, the subject is tried to be discovered through the examples given by the teacher. However, in RME, the learning process is realised as a problem-solving process. According to Realistic Mathematics Teaching, the teacher does not constantly give examples to students. On the contrary, students are expected to find suitable examples for the given situation.

## **Conclusion**

On the foundation of RME, various sources of data such as cultural and social life, the physical environment, history, geography, and folk literature are used to create a meaningful context for learning (Altun, 2006, p. 235). In the teaching process in this approach, Lange (1996), William (1997), and Majewska (2019) aim to establish connections with daily life activities and use materials related to the teaching

process. The cultural richness of nations provides opportunities to create learning environments that encourage students to acquire and rediscover formal mathematical concepts. Research has shown that RME has positive effects on both students and teachers, including increased interest and appreciation for mathematics, improved mathematical proficiency, enhanced problem-solving skills, and increased student motivation (İnci, 2023). Increasing the number of studies and directing teachers and education professionals towards RME is believed not only to enhance mathematical achievement but also to positively influence attitudes towards mathematics (Bintaş et al., 2003; Çilingir, 2015; Gravemeijer, 2020). This approach aims to guide students in viewing mathematics as a tool for out-of-school activities and to help them construct mathematical meanings based on concrete experiences and intuitions (Göğün, 2009).

Due to the abstract nature of mathematics, the mathematics curriculum focuses on conceptual and procedural knowledge, encouraging abstract thinking rather than concrete real-life models. Learning mathematics involves more than acquiring basic skills; it includes thinking critically, understanding problem-solving strategies, developing a positive attitude, and realizing the significance of mathematics as an essential tool in real life (Göğün, 2009, p. 11). Current curricula emphasize that mathematics is learnable for everyone and aim to cultivate individuals with high-level thinking skills. The problem-based learning approach encourages students to be more active participants in solving mathematical problems. RME is a model that gives students the opportunity to apply mathematical concepts in daily life. Therefore, teachers should be provided with support to enhance their professional development in RME strategies. Preparing RME-appropriate problems and investigating whether these problems are compatible with existing evaluation types is also important. The studies show that RME gives positive results when used as a long-term approach. Therefore, it is recommended to prepare more RME-appropriate lesson plans and extend the duration of these lessons.

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#### Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of paper titled “**Realistic mathematics education**” belongs to the authors.