

The Eye-Tracking Method in Mathematics Education

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Introduction

Eye tracking is a process to determine and evaluate where, how long, and how often an individual looks at any given moment, and in what order the eye moves from one direction to another (Duchowski, 2017; Holmqvist et al., 2011; Majaranta & Bulling, 2014; Schall & Bergstrom, 2014). Therefore, the eye-tracking (ET) method offers eye movement data about the areas of the screen or object that people pay attention to, the elements they ignore, and the things they are uncomfortable with (Russell, 2005). The data assist in the interpretation of the individual's interests, visual attention, and perception of the scene (Duchowski, 2017; Horsley, 2014). Therefore, the ET method is used to reveal the cognitive process resulting from the relationship between eye fixations and visual attention (Sennersten, 2008). In other words, the ET method is used to collect data in the context of conducting experimental studies on human perception, cognition, and behavior (Abrahamson et al., 2017; Shayan et al., 2017).

The ET method reflects the cognitive process instantaneously in the various tasks under consideration (Just & Carpenter, 1976; Rayner, 1998). By observing and interpreting eye movements, information is obtained about cognitive operations and processes in the brain (Biedert et al., 2009). There are basically two types of eye movements: fixation and saccade (Salvucci & Goldberg, 2000; Winke et al., 2013). Fixation movement generally refers to situations in which the eye is fixed and immobile in one area (Schall & Bergstorm, 2014; Sharif & Maletic, 2010). The fixation movement provides information about where and for how long the individual looks, that is, where they direct their attention (Rayner, 1998). The fixation duration and fixation count values, derived from this information, can be used to measure attention to the displayed stimulus (Glaholt & Reingold, 2011; Wang et al., 2014b). Fixation duration and fixation count are the most commonly used ET data to assess cognitive processing (Just & Carpenter, 1976). The saccade movement of the eye can be defined as the rapid movement of the eye from one fixation to another in order to help the individual to integrate the whole scene they are looking at (Schall & Bergstrom, 2014).

The data obtained through eye tracking offers researchers information about the processes underlying participants' perception, attention, memory, or reasoning abilities (Maurage et al., 2021). For example, a student's eye fixations can be used to get an idea of the amount of time and attention the student needs to process the given stimulus (Eckstein et al., 2017). Moreover, the accuracy and latency of students' saccadic eye movements can provide information about their cognitive control capacity (Munoz & Everling, 2004) and the count of saccades can provide information about their reasoning ability (Thibaut & French, 2016).

The ET method is founded on the eye-mind theory. According to this theory, the eyes are windows to the mind. That is, eye movements form a highly informative component of people's observable behavior, which allows us to infer what people are thinking based on their actions (Salvucci, 2000). When and where the eyes move in a complex task is the most important factor in the cognitive process during the task (Just & Carpenter, 1980; Rayner et al., 2012). Thanks to the eye-mind theory, it becomes possible to make inferences about a person's intellectual and cognitive process by evaluating the data on where a person looks, where they focus while looking, or how often they look.

The utilization of the ET method offers various benefits to researchers. First of all, it allows the decomposition of tasks into phases with different processing times (Just & Carpenter, 1976). At the same time, the ET systems used in today's technology automatically and in real-time track where the individual is looking on the computer screen (Oyekoya & Stentiford, 2004). These features provide a great advantage to the researcher. In this way, the researcher has an idea about the cognitive process during the execution of the task without being exposed to participant interpretation. In addition, the fact that ET data support other types of data is a great opportunity for researchers (Karn et al., 1999).

Eye trackers

Historically, it can be said that the first techniques used to obtain eye movements were those involving direct physical contact with the cornea. These techniques are potentially harmful methods as they may damage the cornea of the participant (e.g. Javal, 1878). For this reason, in later studies (e.g. Javal, 1879), the eye movements of the participants were analyzed with the help of mirrors. The number of blinks was obtained by means of microphones placed in the eye. However, this method records eye movements subjectively and is limited by the accuracy and memory of the observer (Dodge, 1906). Therefore, in the following periods, new methods were needed to record eye movements objectively and without interfering with the participant.

Many techniques have been tried and developed in the historical process. As of the 2000s, ET devices (eye trackers) have become more ergonomic and capable of faster processing. These devices are based on a method called corneal reflection (Carter & Luke, 2020). That is, they determine the points at which the participant is looking by recording pupil movements (Duchowski, 2017; Lilienthal & Schindler, 2019). Most of these modern eye trackers are video-based and are categorized into two groups: screen-based and mobile (Carter & Luke, 2020). Screen-based eye trackers are ET tools that can be mounted on desktop and laptop computers or used with devices such as tablets and phones through an apparatus (Duchowski, 2017). With these eye trackers, participants' eye movements on the stimuli presented on the screens of devices such as computers and tablets are monitored and recorded. Mobile eye trackers are portable and glasses-like devices that can be worn on the participants' heads (Duchowski, 2017). With these eye trackers, participants' eye movements are monitored and recorded during a physical activity (lecturing, performing a task, etc.). Therefore, while screen-based eye trackers restrict the movements of the participants, mobile

eye trackers offer an environment where participants can move more freely. However, it can be said that the data quality and sampling frequency of mobile eye trackers are relatively lower than screen-based eye trackers (Strohmaier et al., 2020).

Eye-tracking studies in mathematics education

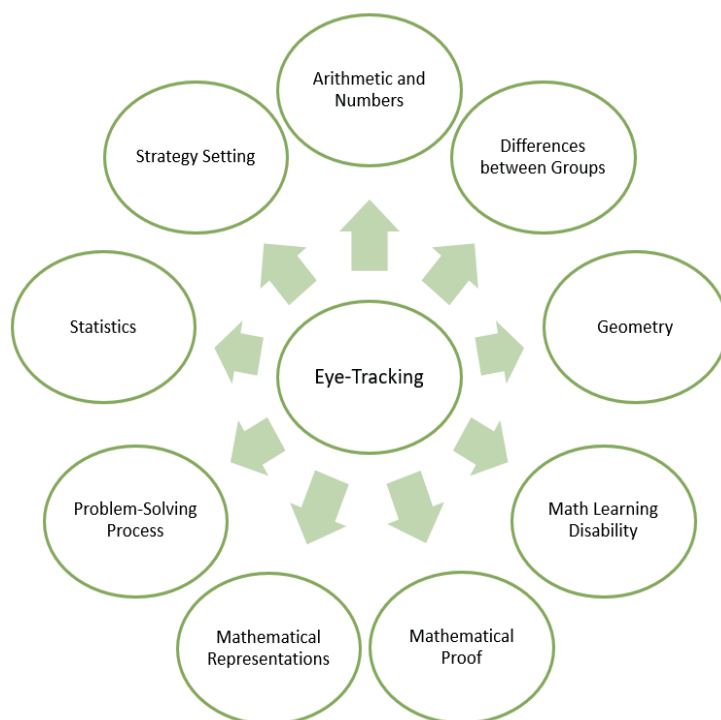
Since the ET method is experimental, a laboratory environment is needed to carry out the studies in a proper way. This causes the studies to be limited to university-level students in general and to be difficult to apply to different audiences. Strohmaier et al. (2020) found that the majority of the studies conducted in mathematics education involved participants from higher education levels.

The ET method provides an idea about where and for how long students look at a stimulus (Duchowski, 2017). However, ET data do not provide information on the cause of these gazes. Therefore, it does not provide a context for researchers. For this reason, using ET data alone is difficult to interpret.

Despite these challenges, eye tracking is a powerful tool for mathematics education research (Schindler & Lilienthal, 2018). The ET method provides a way to make powerful inferences about the difficulties students face in understanding cognitive tasks covering visual, textual, graphical, and stereotypical contents (Obersteiner & Tumpek, 2016; Paskovske & Kliziene, 2024). ET, which provides insights into students' mathematical thinking and learning processes, is a valuable tool in mathematics education research (Strohmaier et al., 2020). Therefore, the use of the ET method in mathematics education research has been increasing in recent years (Paskovske & Kliziene, 2024; Schindler & Lilienthal, 2018; Strohmaier et al., 2020; Türkoğlu & Yalçınalp, 2024).

The ET method can be used in many areas of mathematics education. Some of them are as follows: arithmetic and numbers, differences between groups, geometry, math learning disability, mathematical proof, mathematical representations, problem-solving process, statistics, and strategy setting (Figure 1).

Figure 1. *Usage Areas of the ET Method in Mathematics Education*



Arithmetic and numbers: Arithmetic and numbers constitute the basis of mathematics. Therefore, many ET studies have been conducted on these subjects in mathematics education. Mental calculation (Porrás et al., 2024), mathematical operations (Klein et al., 2014), number line (van't Noordende et al., 2016), fraction comparison strategies (Obersteiner & Tumpek, 2016), calculation estimation (Ganor-Stern & Weiss, 2016), and numerical cognition (Mock et al., 2016) are examples of studies conducted in the field of arithmetic and numbers.

Differences between groups: Determining the differences between groups allows for a comparative analysis of the cognitive characteristics of the groups. Many studies have been conducted to reveal the differences between certain groups when solving the same mathematical problems using eye tracking. For example, differences between eye movements on specific tasks have been examined in groups of students such as strong and weak students in mathematics (Rottman & Schipper, 2002), gifted and non-gifted students (Sajka & Rosiek, 2015), novice and expert students (Wang et al., 2014a), successful and unsuccessful problem solvers (Lin & Lin, 2014), students with high and low mental rotation ability (Roach et al., 2016), and students with and without mathematics learning difficulties (Schindler et al., 2020; van der Weijden et al., 2018). Many variables such as attention span, regions of attention, number of fixations, task completion time, etc. can be compared between groups.

Geometry: Due to the relationship between geometric thinking and visual perception, it has become an appropriate method to use the ET method in the perception and processing of geometric objects (Strohmaier et al., 2020). Therefore, many ET studies have been conducted in the field of geometry. In geometry education, ET studies are carried out on geometric proofs (Alqassab et al., 2018; Schindler et al., 2016), mental rotation tasks (Roach et al., 2016; Voyer et al., 2020),

construction of geometric objects (Lee & Wu, 2017; Shvarts, 2018), the perception of objects in the Cartesian plane (Shvarts, 2018), identification of geometric shapes (Dreneva et al., 2021; Verdine et al., 2017), geometric misconceptions (Uygun et al., 2024), pattern recognition (Baumanns et al., 2024), geometric problem solving (Schindler & Lilienthal, 2019), digital task design (Bairral & Aldon, 2024).

Math learning disability (Dyscalculia): Studies show that eye tracking has diagnostic potential for detecting mathematical difficulties (Almfjord & Hallberg, 2021). In addition to diagnostic studies on mathematics learning disabilities (van Viersen et al., 2013), ET studies are also conducted on basic number processing deficits (Moeller et al., 2009), quantity recognition-comparison (Pitta-Pantazi et al., 2024; Schindler et al., 2019), counting (Schindler et al., 2020), number line (Landerl et al., 2017; Simon & Schindler, 2022).

Mathematical proof: Understanding and reading mathematical proofs is an important activity in learning mathematics (Mejia-Ramos & Inglis, 2009). In literature, there are many studies on mathematical proofs using the ET method. How to pay attention to additional visuals while reading mathematical proofs (Beitlich et al., 2014), proof verification behavior (Inglis & Alcock, 2012), and comparison of reading behaviors in the proofreading and verification process (Panse et al., 2018) can be given as examples of studies conducted in the field of mathematical proof.

Mathematical representations: Mathematical representations are defined as structures that replace, embody, and encode mathematical ideas or relationships (Goldin, 2014). Therefore, mathematical representations have an important place in mathematics education. In teaching of mathematical concepts, it is important to carry out studies on which of the different representations of mathematical concepts is more interesting and which representation facilitates the learning of the concept, etc. In the literature, there are many studies conducted using the ET method on mathematical representations. ET studies on mathematical representations are carried out in many areas such as the ability to read different representations of the mathematical object (Andrá et al., 2015), the effect of visual representation on the understanding of symbolic representation (Atagi et al., 2016), the comparison of representations in terms of comprehensibility and applicability (Bolden et al., 2015), the comparison of the use of single and multiple representations in problem solving (Liang & She, 2021; Malone et al., 2020; Ott et al., 2018).

Problem-solving process: The tracking of students' eye movements during the process of problem-solving allows for the identification of the temporal sequence of solution steps, as well as the acquisition of information pertaining to the underlying problem-solving processes (Schindler et al., 2016). Straka et al. (2021) also state that the ET method is effective in examining students' problem-solving processes, comprehension strategies, and metacognitive abilities. The ET method reflects the cognitive process instantaneously in the various tasks under consideration (Just & Carpenter, 1976; Rayner, 1998). Therefore, it provides researchers with the advantage of uninterrupted observation of the individual's problem-solving process (Inglis & Alcock, 2012; Obersteiner & Tumpek, 2016). This method makes it possible to observe the process without

comment or intervention. In addition, it reveals students' unconscious cognitive processes during problem solving. Even if the student is not aware of these processes, the process can be read from the student's eye movements (Schindler & Lilienthal, 2019). There are many ET studies on the problem-solving process in mathematics education (For example, Lin & Lin, 2018; Mayer et al., 2023; Schindler & Lilienthal, 2019). In addition to these, there are also studies in which the teacher's gaze behaviors throughout the problem-solving activity were examined. For example, Haataja et al. (2018) examined the teacher's gaze behavior when supporting peer interaction and mathematical thinking throughout a collaborative problem-solving activity.

Statistics: The use of the ET method in statistics education studies is relatively less (Boels, 2023; Strohmaier et al., 2020). In the field of statistics education, ET studies on how statistical graphs such as histograms (Boels et al., 2024; Lyford & Boels, 2022), box plots (Abt et al., 2023) and dot plots (Boels, 2023; Schreiter & Vogel, 2024) are analyzed and interpreted. In addition, studies on Bayesian reasoning (Cohen & Staub, 2015) and misconceptions (Fleig et al., 2017) are also carried out.

Strategy setting: Eye tracking is a valuable tool for identifying strategies used in mathematical tasks (Beitlich & Obersteiner, 2015; Obersteiner & Tumpek, 2016). The ET method is particularly suitable for analyzing problem-solving scenarios involving visual strategies (Malone et al., 2020; Ott et al., 2018). In the literature, there exist many ET studies performed to determine the strategies used by students while solving mathematical problems (Kang et al., 2023; Lin & Lin, 2014; Zheng & Cook, 2012).

Recommendations for eye-tracking research

The ET method is becoming increasingly important in the field of mathematics education. Since the method requires an experimental application and is relatively new, it is recommended to pay attention to some issues. Therefore, under this heading, some suggestions that may help researchers in ET research are presented.

a) Supporting eye-tracking research with think-aloud data

The ET method, which assumes that there is a connection between an individual's visual focus and cognitive focus (Just & Carpenter, 1980), suggests that eye movements are related to the cognitive process. However, the ability to accurately associate eye movements with cognitive processes is one of the most important challenges in ET research (Strohmaier et al., 2020).

ET data do not provide context for the researcher. Interpretation of eye movements without providing context to the data may not always be correct. For example, the fixation of the participant at a point for a long time may be interpreted as the participant finding that area interesting, but it may also mean that the participant has difficulty interpreting that area. For this reason, ET data should be combined with other methods and data that provide additional information about the participants' experiences (Schindler & Lilienthal, 2019; Tobii Technology, 2009). It is generally advised that ET data be supplemented with other data collection methods, such as the think-aloud

method (Csíkos & Steklács, 2015; Elbabour et al., 2017; Elling et al., 2011) or interviews (Bolden et al., 2015).

There are two principal types of think-aloud method: concurrent think-aloud (CTA) and retrospective think-aloud (RTA) (Tobii Technology, 2009). In the CTA method, students explain their thoughts aloud at the same time while completing the task. This shows that the CTA method may change what is thought during the task; that is, it is reactive. Compared to the RTA method, the CTA method is more susceptible to reactivity (Russo et al., 1989). Since reactivity may negatively affect the validity of the studies (Russo et al., 1989), it may be recommended to use the RTA method instead of CTA in ET studies.

In the RTA protocol, the participant may not remember everything he/she thought during the task. This may lead to participants' thoughts being expressed less, incompletely, or restructured, that is, as fabricated (Elling et al., 2011; Guan et al., 2006; Russo et al., 1989). Furthermore, verbal accounts may contain a greater degree of rational judgment or strategy than the actual judgments and strategies employed by participants when undertaking the test. The participant may clarify their actions in a way that makes them appear more systematic, rational, organized, thoughtful, or coherent (Kuusela & Paul, 2000). This shows that the RTA protocol may not be accurate in this context (Russo et al., 1989).

In order to prevent the accuracy problem encountered in the RTA method and to help the participant's memory, a cued RTA method is proposed. In the cued RTA method, participants are instructed to think aloud retrospectively based on a record of observations or intermediate products in the problem-solving process and are reminded of their memories of this process (Van Gog et al., 2004). Using a supporting video or other memory cues helps to recall the information in long-term memory (Hyrskykari et al., 2008). This leads to better results as it reduces forgetting and/or fabrication of thoughts (Van Someren et al., 1994). Cued RTA methods can be diversified as video-cued, gaze graph-cued, gaze video-cued.

The problem of not remembering the thoughts correctly (accuracy problem) encountered in the RTA method is solved by using the combination of ET and think-aloud methods together (Elling et al., 2011). The use of eye movements as cues in RTA methods allows participants to produce more words and comments (Tobii Technology, 2009). Many studies also argue that adding eye cues to the think-aloud method is beneficial and provides more informative and multi-word verbal data (Ball et al., 2006; Eger et al., 2007; Hyrskykari et al., 2008; Olsen et al., 2010; Russo, 1979). In the RTA method with ET cues, participants are shown the real behavior by watching their own gaze again as a video. Since it reduces fabricated verbal expressions, it is considered as a valid and reliable method in ET research (Guan et al., 2006).

b) Spaced presentation of stimuli

The first fixation point shows where the students start to solve the task. It has an important role in determining the behavior pattern performed during the solution of a task. Therefore, determining

the first fixation point correctly is important for the reliability of the study.

A saccade is influenced by the properties of the immediately preceding fixation and saccade (Tatler & Vincent, 2008). When stimuli are presented to students in succession, the first fixation point for a stimulus may be the same as or close to the last fixation point of the previous stimulus. Therefore, the first fixation point will be affected by the previous stimulus, and reliable data will not be obtained. In order to prevent this and to correctly identify the first fixation point, it may be appropriate to show a blank page between stimuli. In this way, the student will have the opportunity to rest his/her eyes between the two stimuli, and the factor that will affect the first eye movement of the student will be eliminated.

c) Training of participants

One of the situations affecting the reliability of the ET studies is, conducting with the participants who encountered the ET method for the first time. Since novice participants who are not familiar with the ET method will try to get used to the method during the application, reliable eye recordings may not be obtained. In order to prevent this and increase the reliability of the data, it is recommended to train the participants (Elbabour et al., 2017; Elling et al., 2011). A training session can be organized for students to experience the environment that they will encounter in real practice within the scope of the research. This session can be in the form of training in which the ET method is explained, or it can be realized by the student experiencing the ET method with a different stimulus. In this session, students will gain experience on how the ET device works, how to calibrate/decalibrate, how far they should stand from the screen, and their positions. Therefore, the quality and reliability of the data obtained from ET devices can be increased.

d) Data quality

ET data quality is generally considered in terms of accuracy and sensitivity. Accuracy is defined as the systematic shift between a known target position and the gaze position reported by the eye tracker (Hessels et al., 2015). Sensitivity is defined as the ability to reliably reproduce a measurement (Nyström et al., 2013). Therefore, obtaining high-quality ET data is important for the accuracy and repeatability of the study. However, Strohmaier et al. (2020) found in their review that most of the ET studies in mathematics education do not adequately mention eye movement data quality.

An additional measure of ET data quality is data loss (Porrás et al., 2024). Data loss may occur due to the student's movement, sneezing, etc., or due to the fact that the calibration is disturbed, or the environment is noisy, bright, etc. Especially when using screen-based eye trackers, it is extremely difficult to avoid data loss, given that some samples are always lost due to participants blinking or averting their gaze away from the monitor (Tobii, A. B., 2023). Therefore, it is important to report the data losses that occur in the studies.

Calibration is required to ensure the accuracy of the data. With this process, the point that the student looks at and the point detected by the device are matched. Although the calibration

process is vital for ET research, the vast majority of ET studies in mathematics education do not mention the calibration procedure (Strohmaier et al., 2020). When performing long-term tasks, performing multiple calibrations in the process increases the accuracy of the data (Carter & Luke, 2020). Therefore, it is important for ET studies that long-term tasks are performed in parts and that the calibration is renewed between each part.

e) Organization of the experimental environment

In the most general sense, the ET method records the instant attention of the individual. Thus, for ET studies to be conducted effectively, experimental environment must be optimized by removing distractions. There are many features that the laboratory environment should have during ET experiments. Distraction of the student's attention during the application has a negative effect on obtaining healthy data. For this reason, firstly, the experimental environment should be free from distracting factors such as noise, clutter, or crowds. The second factor affecting ET studies is related to lighting. A brightly lit environment makes it difficult for the eye tracker to perceive the eye reflection. Therefore, a dimly lit environment allows quality data to be obtained. Thirdly, fixing participant position is an important factor for ET studies. Participants are not allowed to change their position during the experiment in order not to disturb the calibration. For this reason, the height of the students' seats or the distance between the computer and the participant can be regulated so that the participants can sit in a comfortable position (Duchowski, 2017; Płużyczka, 2018).

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